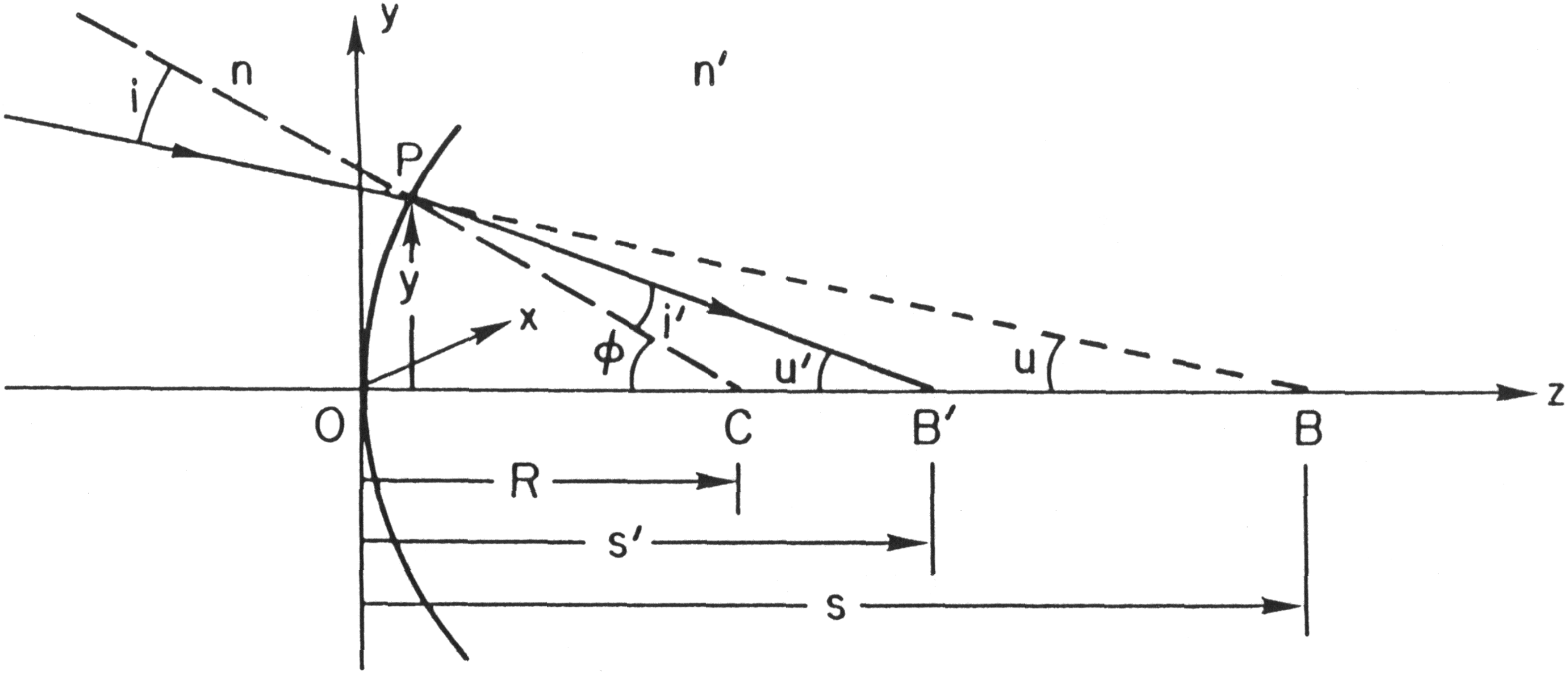
Basic Telescope Optics

Andreas Quirrenbach
Landessternwarte
Universität Heidelberg

Optics and Telescopes

- M. Born, E. Wolf, *Principles of Optics*
- P. Léna, F. Lebrun, F. Mignard, *Observational Astrophysics*
- D.J. Schroeder, *Astronomical Optics*
- R.R. Shannon, *The Art and Science of Optical Design*
- M.J. Kidger, *Fundamental Optical Design*
- R.N. Wilson, *Reflecting Telescope Optics I / II*

Refraction at a Spherical Interface

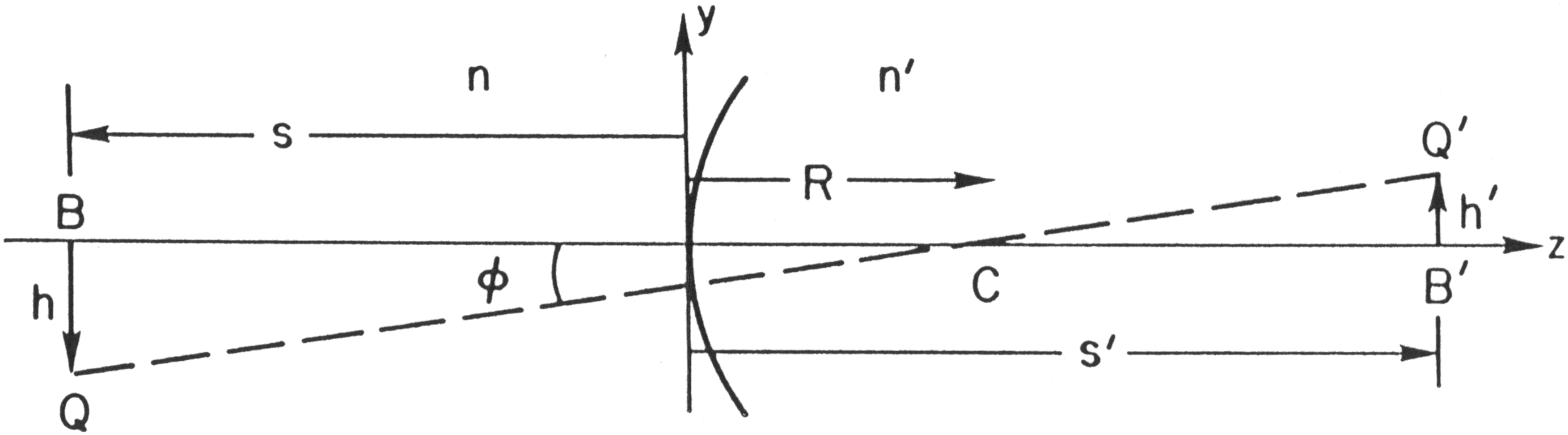


Sign convention: all angles and distances in this diagram are positive

Basics of Paraxial Optics

- Paraxial approximation: y and all angles are small
- Law of refraction: $n \cdot \sin i = n' \cdot \sin i'$, in paraxial approximation $n \cdot i = n' \cdot i'$
- Points at distances s and s' from vertex are called *conjugate points* (image is conjugate to object)
- If s or $s' = \infty$, the conjugate distance is called *focal length*

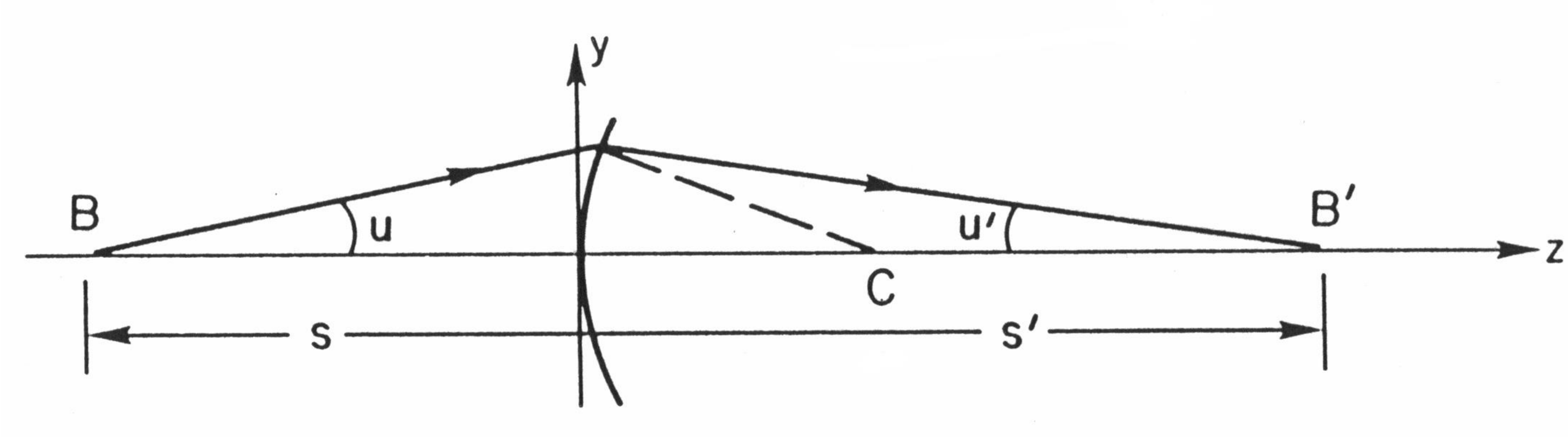
Conjugate Points in the Paraxial Region



B and B', Q and Q' are pairs of conjugate points

Transverse magnification: $m = h'/h$

Angular Magnification

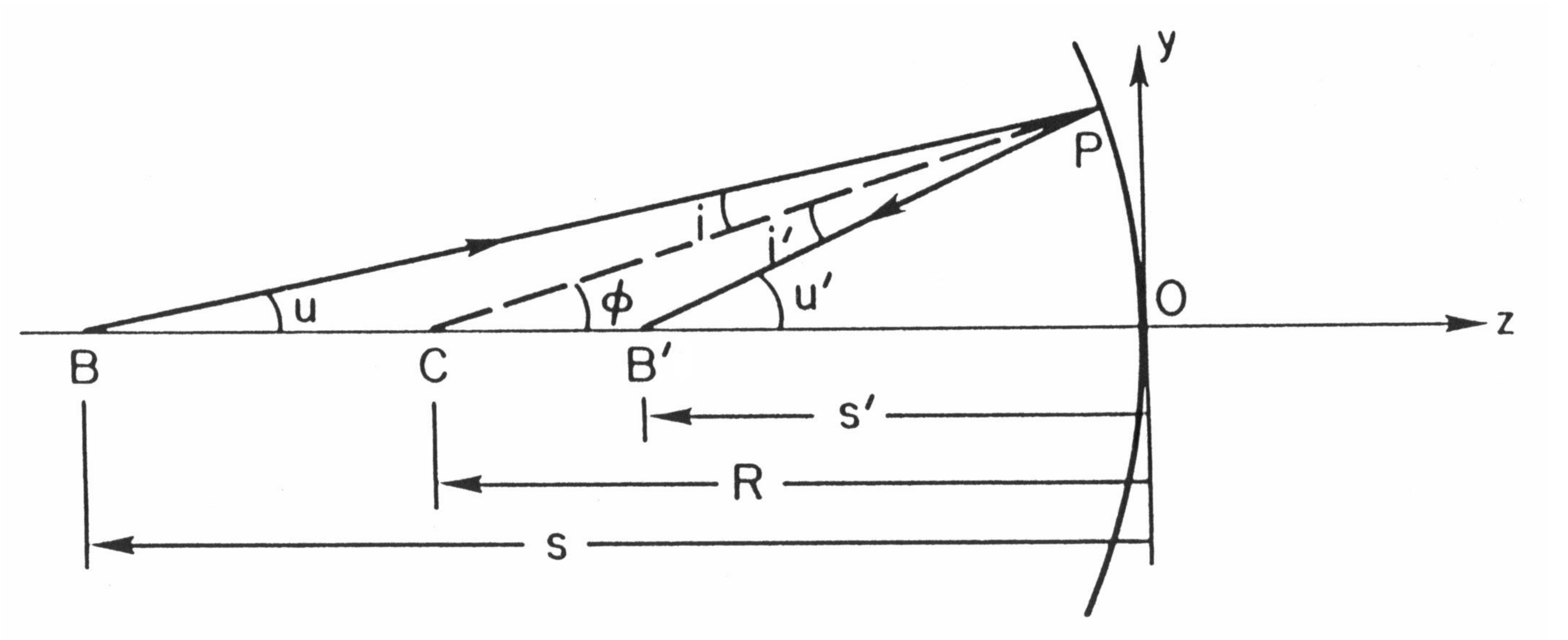


Angular magnification: $M = \tan u' / \tan u = s / s'$

Power, Magnification, Lagrange Invariant

- Definition of power: $P \equiv \frac{n'}{s'} - \frac{n}{s} = \frac{n'-n}{R} = \frac{n'}{f'} = -\frac{n}{f}$
- Transverse magnification: $m \equiv \frac{h'}{h} = \frac{s'-R}{s-R} = \frac{ns'}{n's}$
- Angular magnification: $M \equiv \frac{\tan u'}{\tan u} = \frac{s}{s'} = \frac{n}{n'm} = \frac{nh}{n'h'}$
- Lagrange invariant: $H \equiv nh \tan u = n'h' \tan u'$
- In paraxial approximation: $H \equiv nhu = n'h'u'$

Reflection at a Spherical Surface

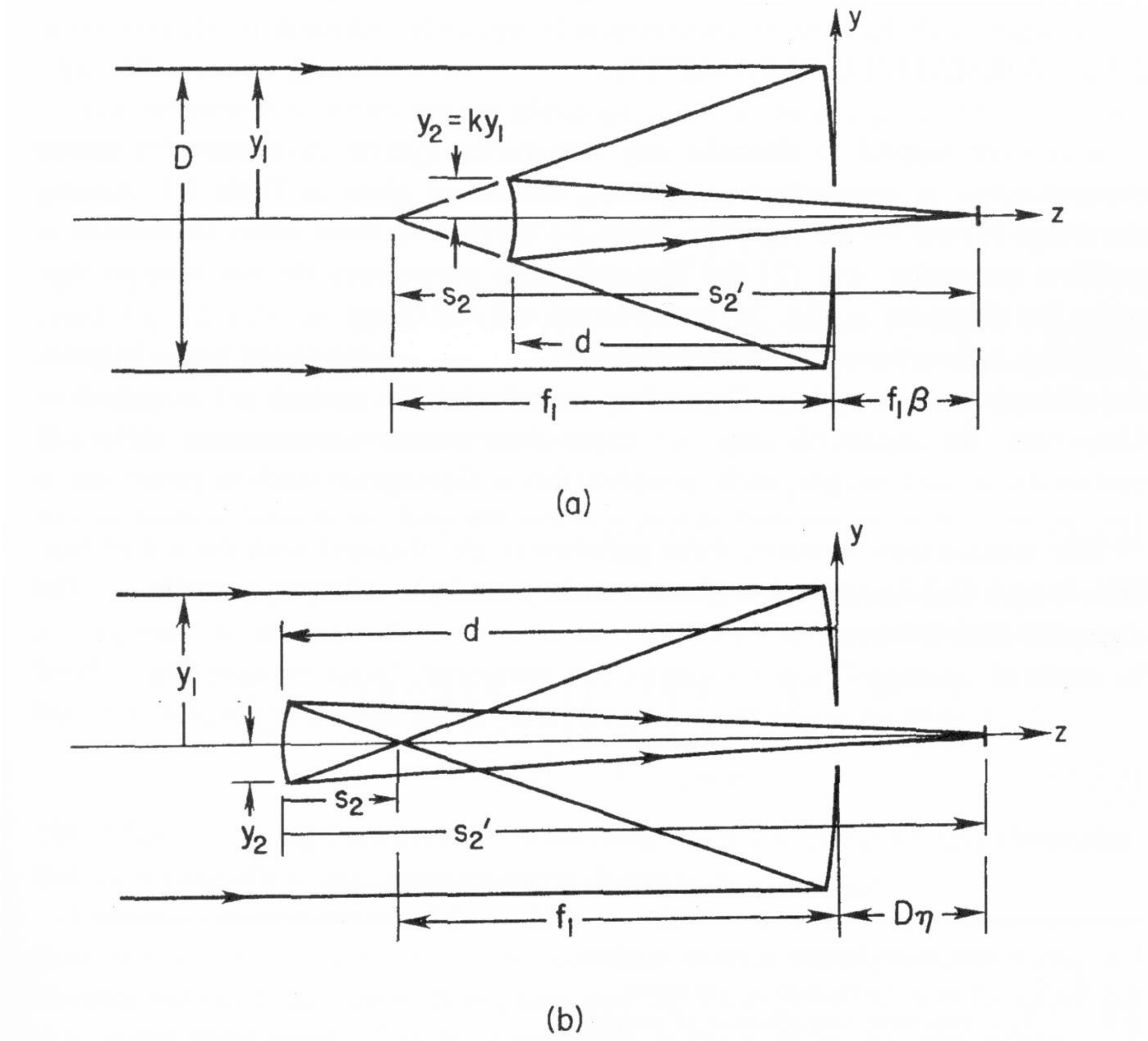


Setting $n' = -1$ for reflection gives unified formulae for lenses and mirrors

Basic Relations for Simple Optical Systems

- Power of two-surface system (thick lens, two-mirror telescope): $P = P_1 + P_2 - \frac{d}{n} P_1 P_2$
- Thin lens (d = 0): $P = P_1 + P_2 = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
- Image scale: $S\left[''/\text{mm}\right] = \frac{206265}{f\left[\text{mm}\right]}$
- Image size: $x\left[\mu\text{m}\right] = 4.86 \cdot f\left[\text{m}\right] \cdot \phi\left['' \right]$
- Focal ratio: $F = f / D$
- Systems with small focal ratio (e.g., f / 1.5) are called “fast” those with large focal ratio “slow”

Two-Mirror Reflecting Telescopes



(a) Cassegrain

(b) Gregorian

Normalized Parameters for Two-Mirror Telescopes

$k = y_2/y_1 =$ ratio of ray heights at mirror margins,
$\rho = R_2/R_1 =$ ratio of mirror radii of curvature,
$m = -s'_2/s_2 =$ transverse magnification of secondary,
$f_1\beta = D\eta =$ back focal distance, or distance from vertex of primary mirror to final focal point,
β and η, back focal distance in units of f_1 and D, respectively,
$F_1 = |f_1|/D =$ primary mirror focal ratio,
$W = (1-k)f_1 =$ distance from secondary to primary mirror,
$=$ location of telescope entrance pupil relative to the secondary when the primary mirror is the aperture stop,
$F = |f|/D =$ system focal ratio, where f is the telescope focal length.

Important Relations for Two-Mirror Telescopes

- Apply standard formulae to the secondary:

$$\frac{1}{s_2'} = \frac{2}{R_2} - \frac{2}{kR_1} = \frac{2}{R_1}\left(\frac{1}{\rho} - \frac{1}{k}\right) = \frac{1}{s_2}\left(\frac{k-\rho}{\rho}\right) = -\frac{1}{ms_2}$$

- Solve for m, ρ, and k in turn:

$$m = \frac{\rho}{\rho - k} \quad , \quad \rho = \frac{mk}{m-1} \quad , \quad k = \frac{\rho(m-1)}{m}$$

- Other relations:

$$1 + \beta = k(m+1) \quad , \quad \eta = F_1\beta$$

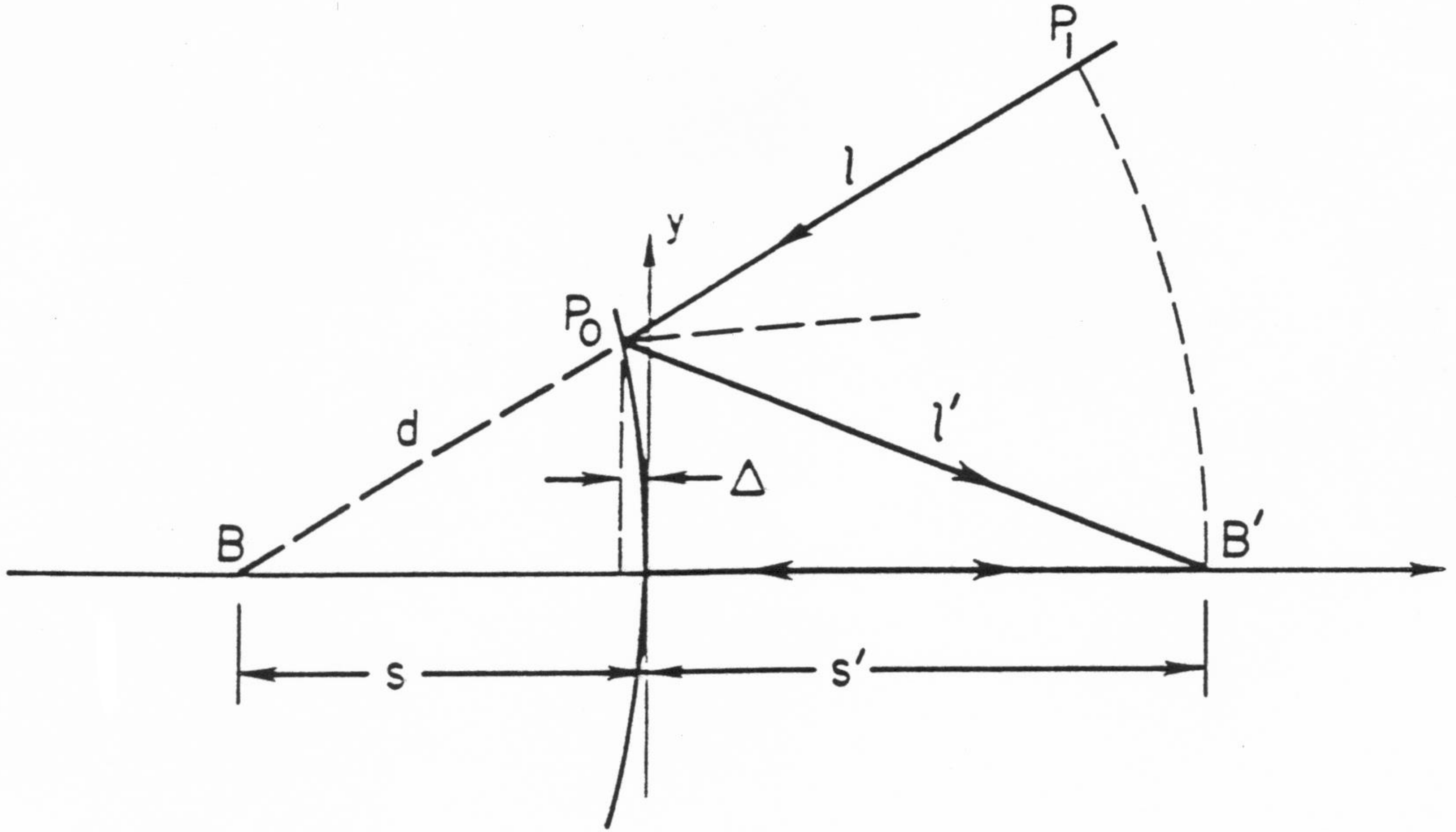
$$P = P_1(1 - k/\rho) = P_1/m \quad , \quad m = f/f_1 = F/F_1$$

Fermat's Principle

- The optical path length of an actual ray between any two points P_0 and P_1 is shorter than the optical path length of any curve which joins these points and lies in a neighborhood of it
- Formulation as variation principle: $\delta \int n\, ds = 0$
- In (y,z) plane: $\delta \int_{P_0}^{P_1} n(y,z)\sqrt{1+y'^2}\,dz \equiv \delta \int_{P_0}^{P_1} F\left(y, y', z\right)dz = 0$
- “Lagrange equation” for Fermat’s Principle:

$$\frac{\partial F}{\partial y} - \frac{d}{dz}\left(\frac{\partial F}{\partial y'}\right) = 0$$

Rays between Conjugates at Finite Distances via Convex Reflector



Derivation of Shape for Convex Reflector (Finite Object Distance)

- Fermat's Principle: $l + l' = 2s'$
- From previous figure:

$$d^2 = y^2 + \left(-s - \Delta\right)^2 \quad , \quad l + d = s' - s \quad ,$$

$$l'^2 = y^2 + \left(s' + \Delta\right)^2 \quad , \quad \Delta = -z$$

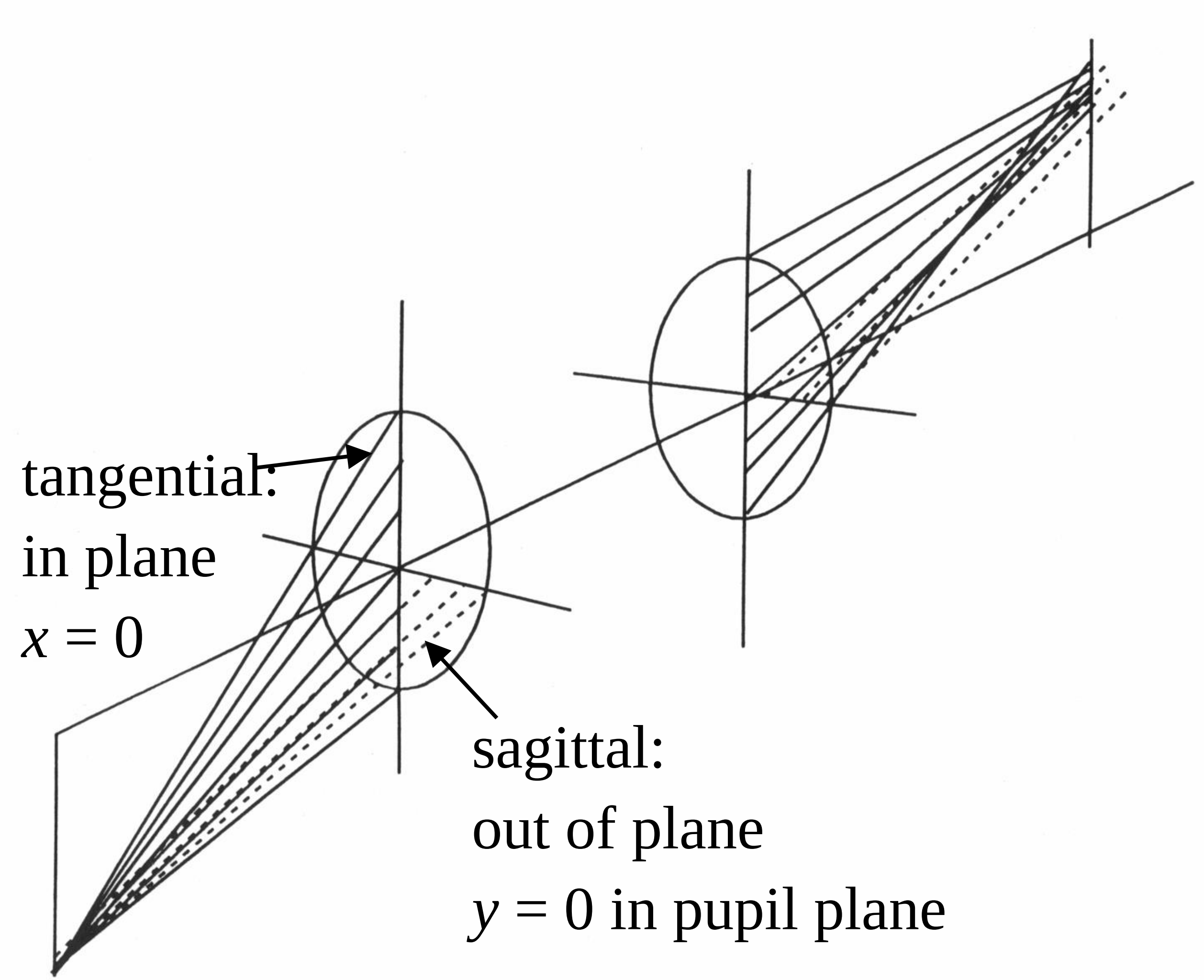
- Some algebra: $y^2 - 4z \frac{ss'}{s+s'} + 4z^2 \frac{ss'}{\left(s+s'\right)^2} = 0$
- Using $\frac{ss'}{s+s'} = \frac{R}{2}$, and defining $1 - e^2 \equiv \frac{4ss'}{\left(s+s'\right)^2}$:

$$\boxed{y^2 - 2Rz + \left(1 - e^2\right)z^2 = 0}$$ (hyperbola, since $ss' < 0$)

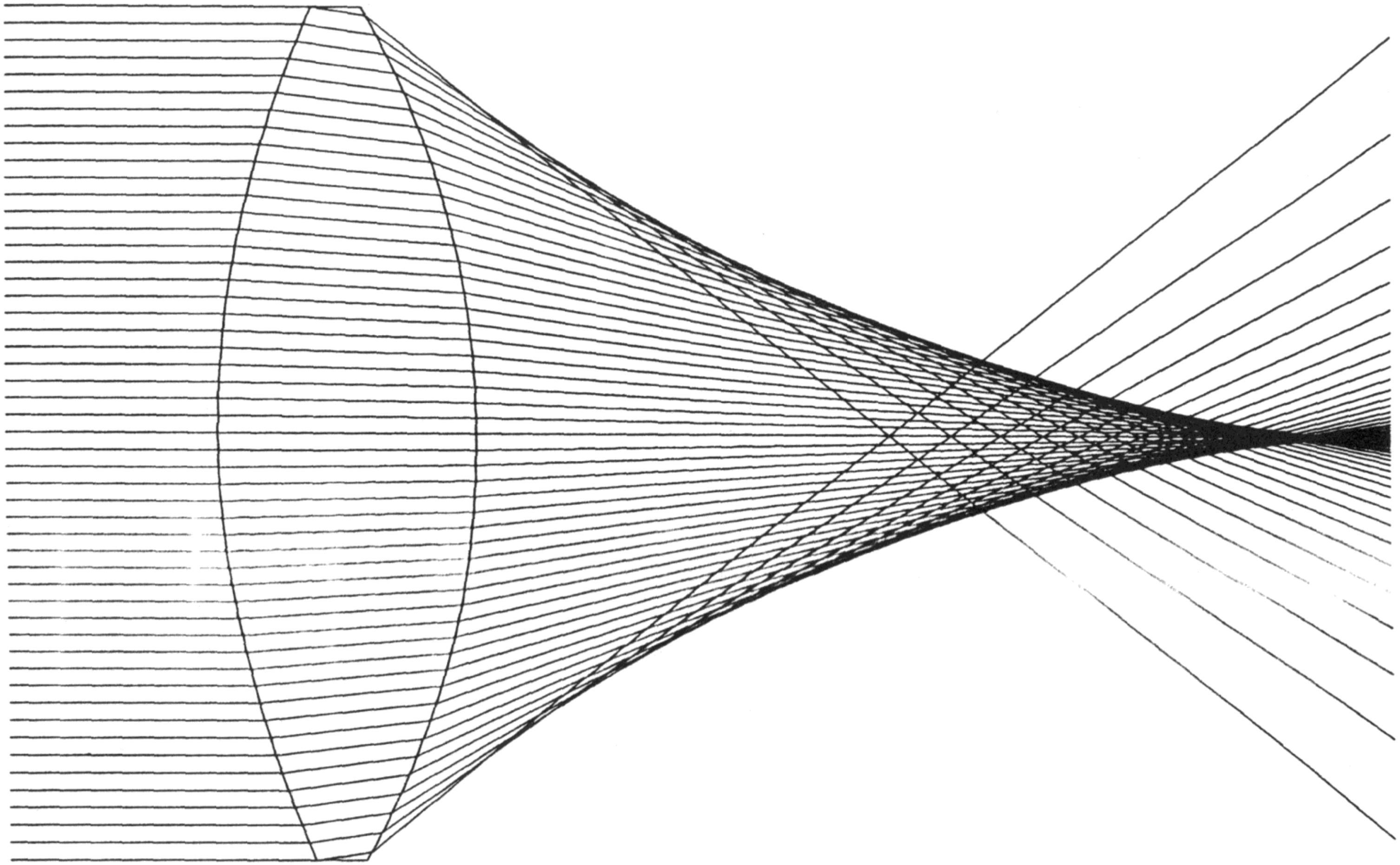
Conic Sections

- General description: $y^2 - 2Rz + (1 - e^2)z^2 = 0$
- Define *conic constant*: $K \equiv -e^2$
  - Oblate ellipsoid: $K > 0$
  - Sphere: $K = 0$
  - Prolate ellipsoid: $-1 < K < 0$
  - Paraboloid: $K = -1$
  - Hyperboloid: $K < -1$

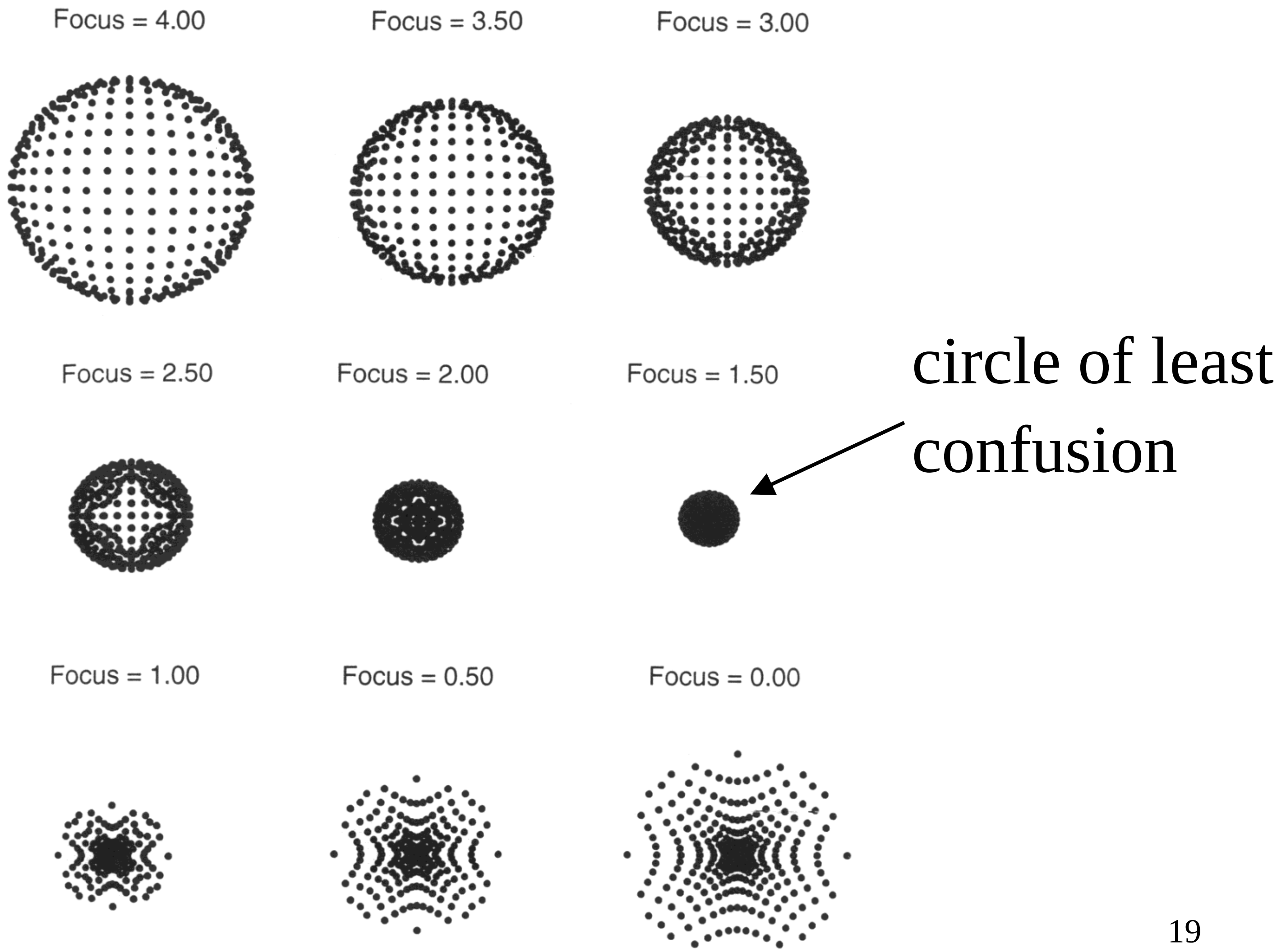
Definition of Sagittal (Dashed) and Tangential (Continuous) Rays



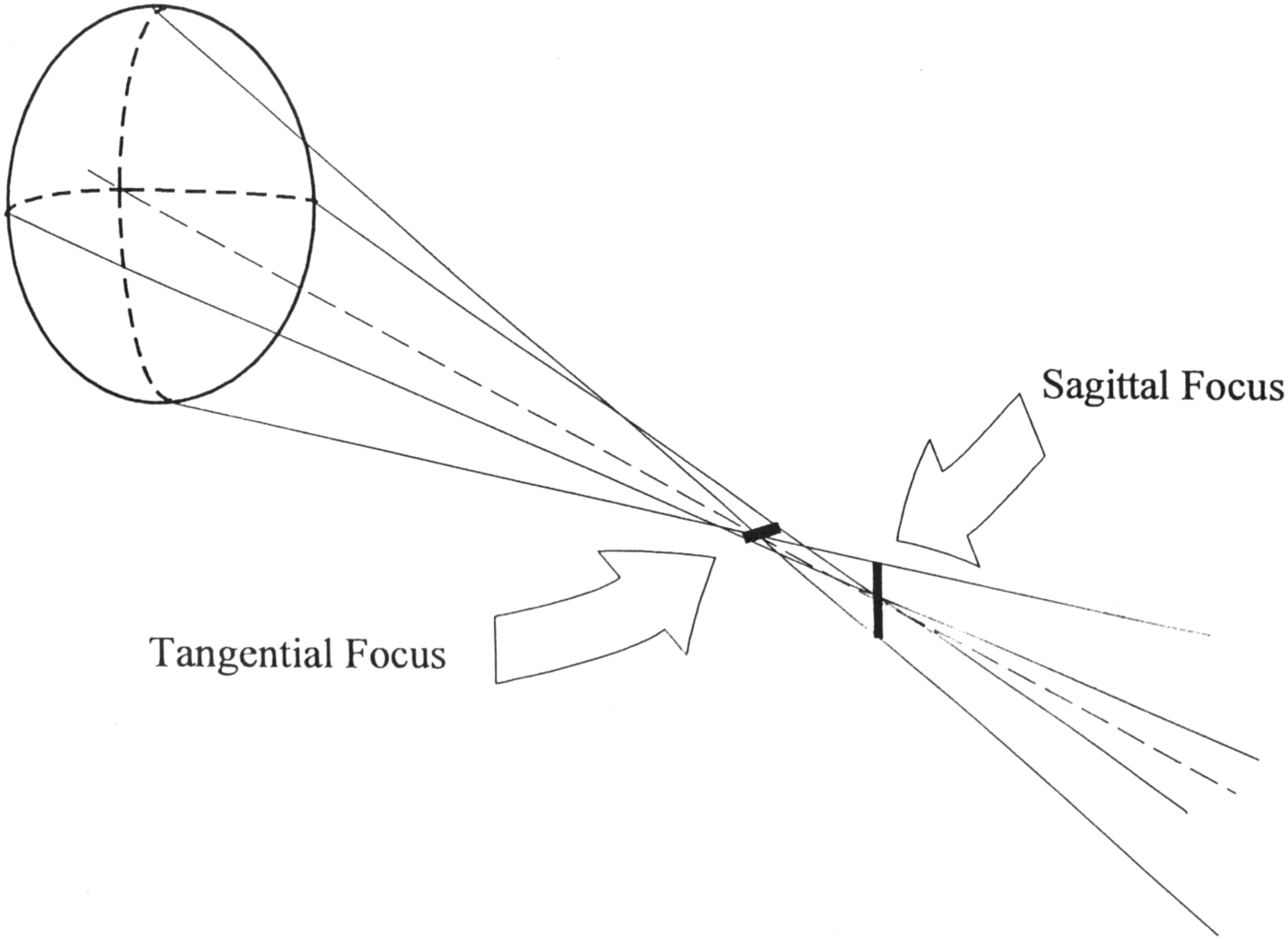
Ray Diagram for a Lens Showing Spherical Aberration



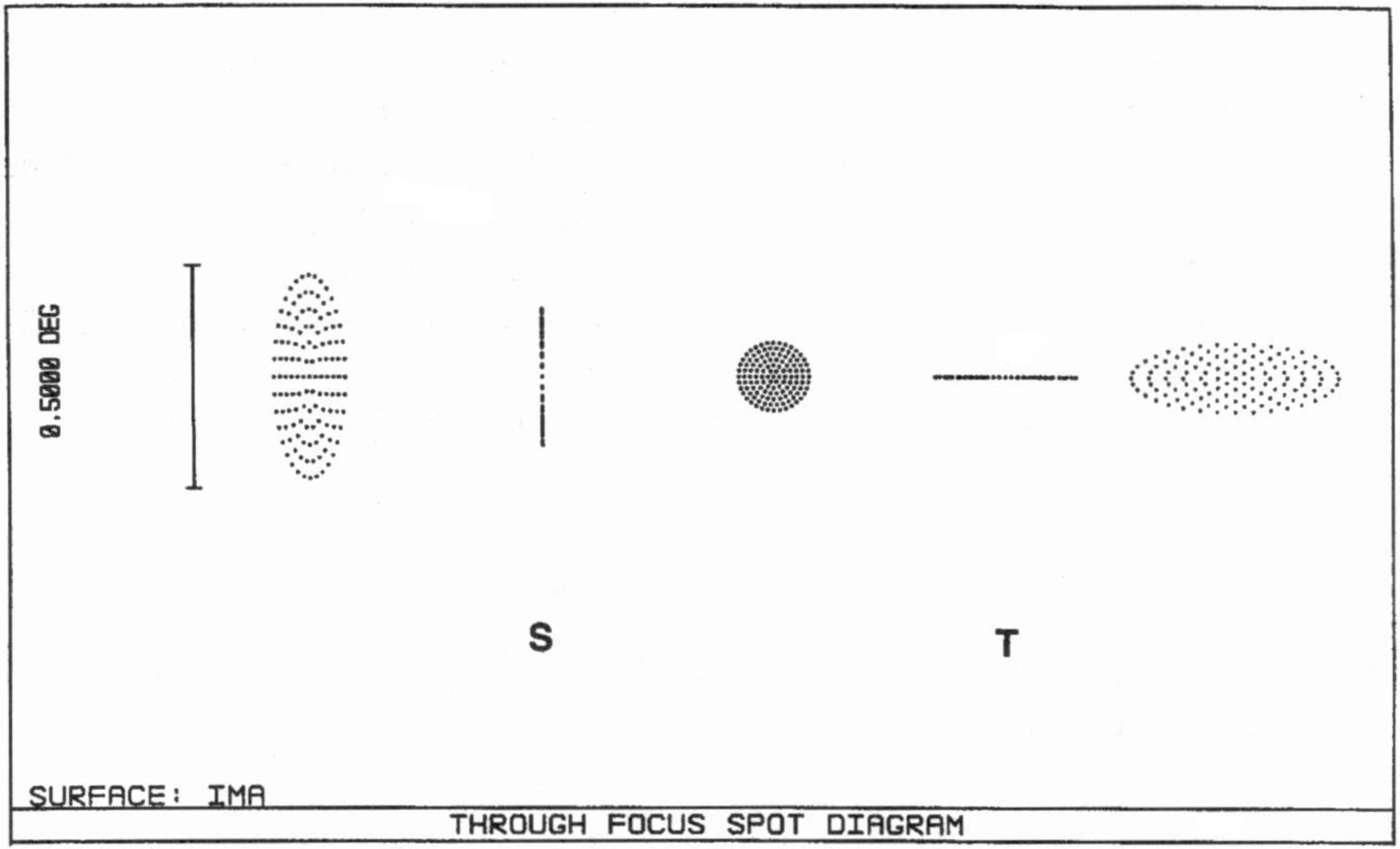
Spot Diagrams through Focus for Lens with Spherical Aberration



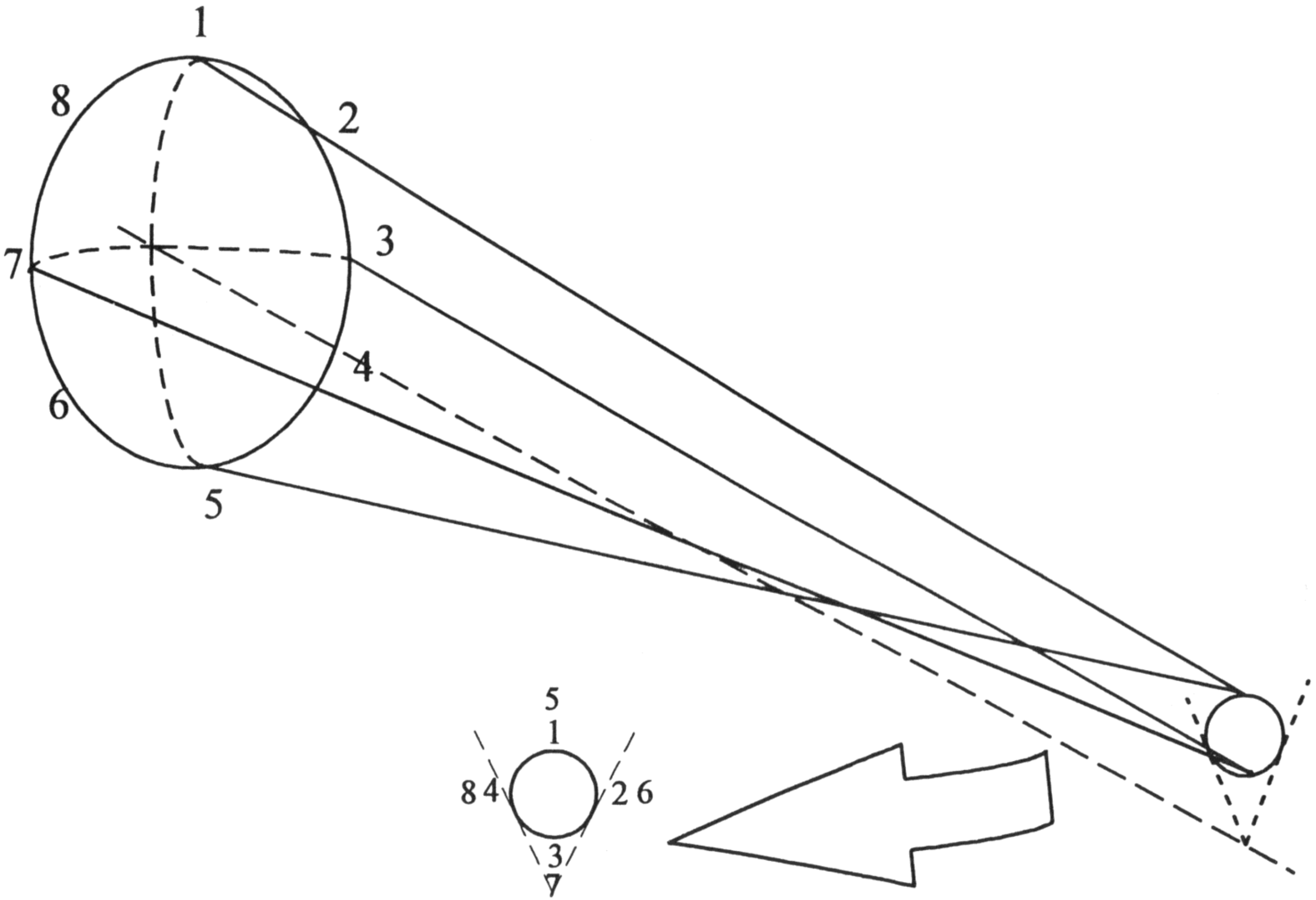
Behavior of Rays in the Presence of Astigmatism



Spot Diagrams through Focus for Lens with Astigmatism

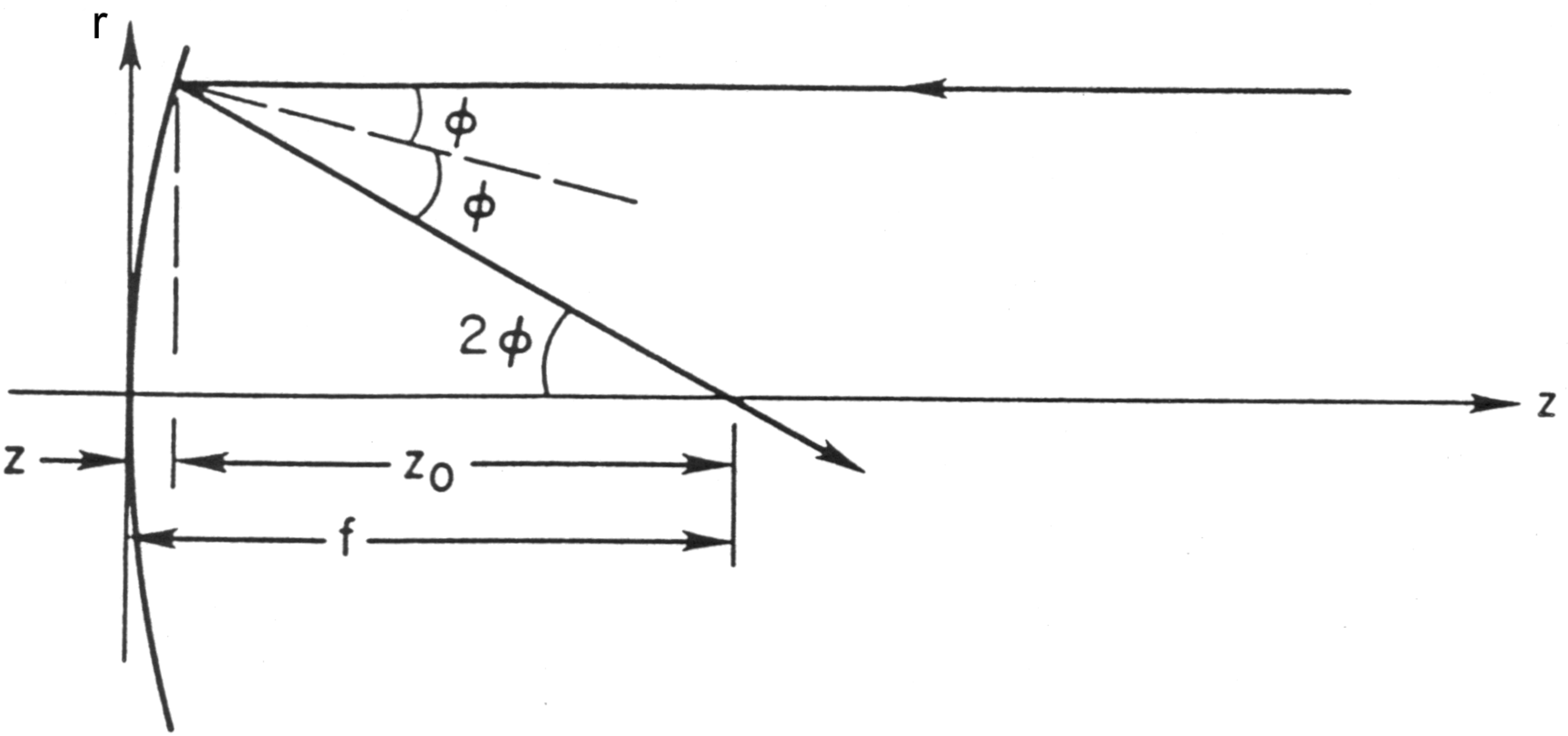


Behavior of Rays in the Presence of Coma



Spot Diagrams through Focus for Lens with Coma

Ray from Distant Object Reflected by Concave Mirror



Focal Length for Rays at Distance *r* from Axis

- From the geometry on the previous viewgraph:

$$z_0 = \frac{r}{\tan 2\phi} = \frac{r\left(1-\tan^2\phi\right)}{2\tan\phi}$$

- For conic sections:

$$r^2 - 2Rz + (1+K)z^2 = 0 \Rightarrow \tan\phi = \frac{dz}{dr} = \frac{r}{R-(1+K)z}$$

- Inserting the second formula into the first:

$$f = z + z_0 = \frac{R}{2} + \frac{(1-K)z}{2} - \frac{r^2}{2\left(R-(1+K)z\right)}$$

Power Series

- Power series for *z* and *f* from binomial series:

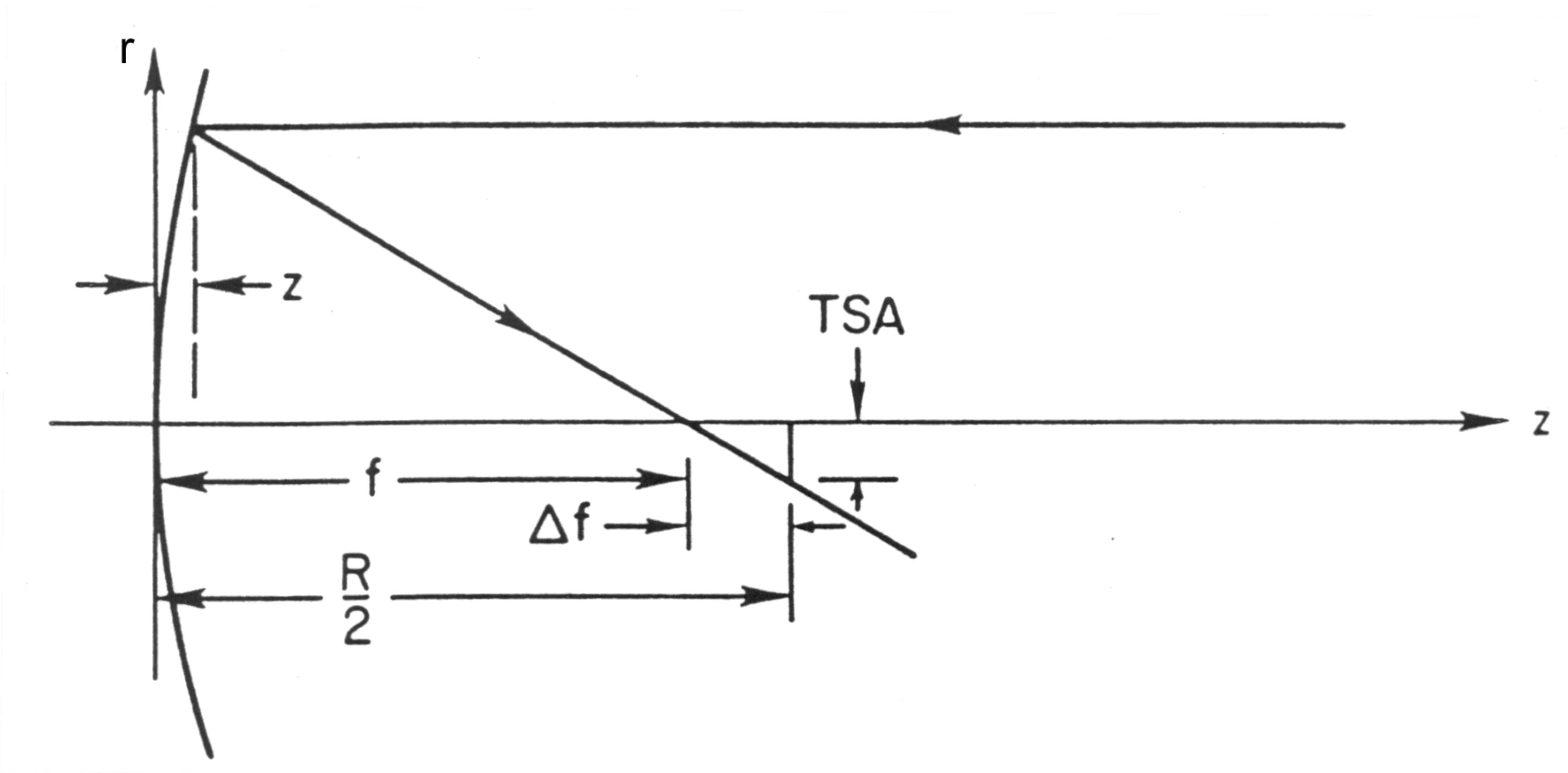
$$r^2 - 2Rz + (1+K)z^2 = 0 \Rightarrow$$

$$z = \frac{R}{1+K}\left[1-\left(1-\frac{r^2}{R^2}(1+K)\right)^{1/2}\right]$$

$$= \frac{r^2}{2R} + (1+K)\frac{r^4}{8R^3} + (1+K)^2\frac{r^6}{16R^5}\dots$$

$$f = \frac{R}{2} - \frac{(1+K)r^2}{4R} - \frac{(1+K)(3+K)r^4}{16R^3} - \dots$$

Transverse Spherical Aberration at the Paraxial Focus



Transverse and Angular Spherical Aberration

- From the figure on the previous viewgraph:

$$\frac{TSA}{\Delta f} = \frac{r}{f - z}$$

- Power series expansion:

$$TSA = -(1+K)\frac{r^3}{2R^2} - 3(1+K)(3+K)\frac{r^5}{8R^4} + \ldots$$

$$= TSA3 + TSA5 + \ldots$$

- Corresponding angular aberration:

$$ASA3 = \tfrac{2}{R}\, TSA3 = -(1+K)\frac{r^3}{R^3} \propto F^{-3}$$

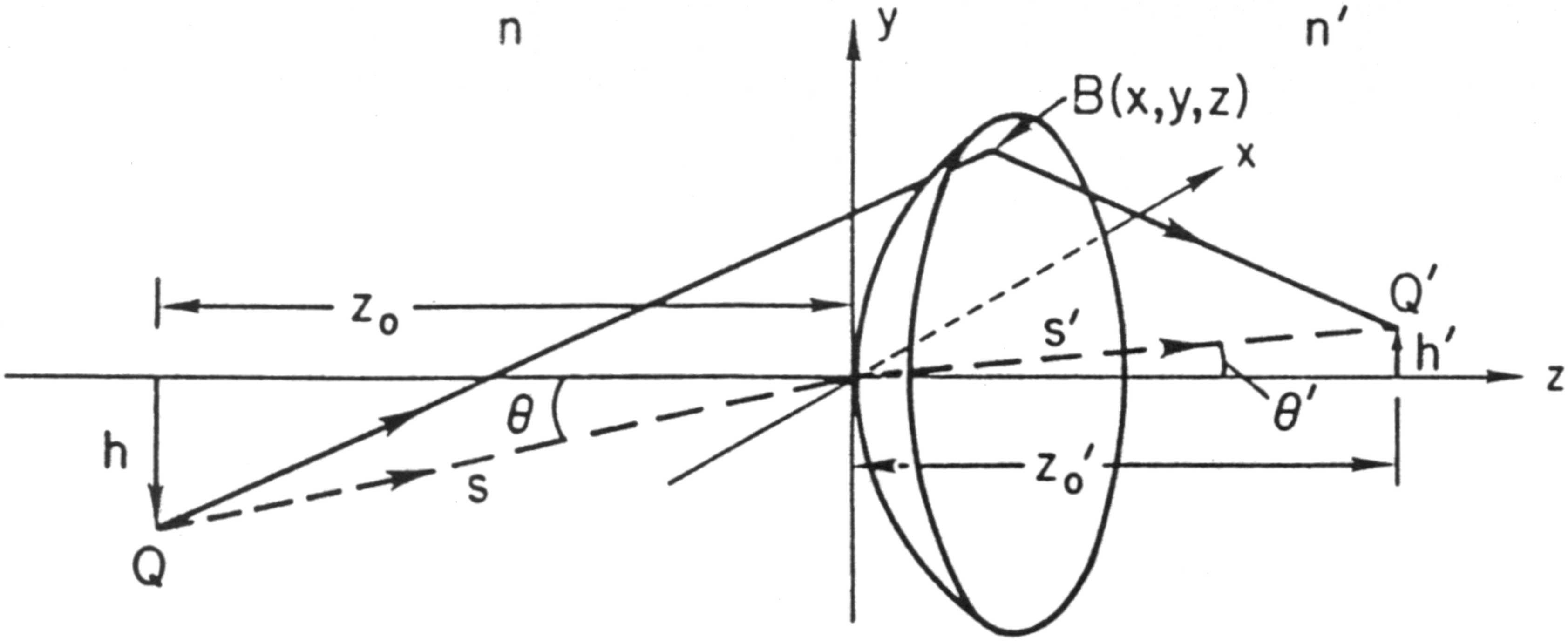
Higher-Order Aberrations

- From the formula on the previous page:

$$\frac{TSA5}{TSA3} = \frac{3(3+K)r^2}{4R^2} = \frac{3(3+K)}{64F^2}$$

- For a sphere with $F = 1.19$, $TSA5$ is 10% of $TSA3$
- Higher-order aberrations are even less important for slower systems
- In most cases considering third-order aberrations is sufficient

Path of Arbitrary Ray through Refracting Surface



Q and *Q*' lie in the *yz* plane; *B* is on the surface
the chief ray passes through the origin

Optical Pathlength through Refracting Surface

$$OPL = \left(-ns + n's'\right)^{\textcircled{1}} - y\left(n'\sin\theta' - n\sin\theta\right)^{\textcircled{2}}$$

$$+ \frac{y^2}{2}\left[\frac{n'\cos^2\theta'}{s'} - \frac{n\cos^2\theta}{s} - \frac{n'\cos\theta' - n\cos\theta}{R}\right]^{\textcircled{3}}$$

$$+ \frac{x^2}{2}\left[\frac{n'}{s'} - \frac{n}{s} - \frac{n'\cos\theta' - n\cos\theta}{R}\right]^{\textcircled{4}}$$

astigmatism

$$- \frac{x^2 y}{2}\left[\frac{n\sin\theta}{s}\left(\frac{1}{s} - \frac{\cos\theta}{R}\right) - \frac{n'\sin\theta'}{s'}\left(\frac{1}{s'} - \frac{\cos\theta'}{R}\right)\right]$$

$$- \frac{y^3}{2}\left[\frac{n\sin\theta}{s}\left(\frac{\cos^2\theta}{s} - \frac{\cos\theta}{R}\right) - \frac{n'\sin\theta'}{s'}\left(\frac{\cos^2\theta'}{s'} - \frac{\cos\theta'}{R}\right)\right]$$

coma

$$+ \frac{r^4}{8}\left[\frac{1}{R^2}\left(\frac{n'}{s'} - \frac{n}{s} - \frac{1+K}{R}\left(n'\cos\theta' - n\cos\theta\right)\right) + \frac{n}{s}\left(\frac{1}{s} - \frac{\cos\theta}{R}\right)^2 - \frac{n'}{s'}\left(\frac{1}{s'} - \frac{\cos\theta'}{R}\right)^2 - \frac{b}{n'-n}\left(n'\cos\theta' - n\cos\theta\right)\right]$$

spherical aberration

① = OPL (chief ray)

② = 0 (Snell's Law)

③ = 0 for tangential astigmatic image

④ = 0 for sagittal astigmatic image

Structure of Optical Path Difference

- Define Φ as optical path difference to chief ray:

$$\Phi = A_0 y + A_1 y^2 + A_1' x^2 + A_2 y^3 + A_2' x^2 y + A_3 r^4$$

- From Φ one can compute the aberrations
  - $|TAS|$ = half-length of astigmatic line image = diameter of astigmatic blur circle
  - $3|TSC|$ = length of comatic flare = 1.5 × width of comatic flare
  - $|TSA|$ = radius of blur at paraxial focus = 2 × diameter of circle of least confusion
  - TDI = distortion

Third-Order Transverse Aberrations for a Mirror Surface

Transverse Aberrations for Mirror Surface[a]

$$\mathrm{TSA} = -\frac{y^3}{R^3}\left[K + \left(\frac{m+1}{m-1}\right)^2\right]s' + \frac{by^3}{2n}s'$$

$$\mathrm{TSC} = \frac{y^2}{R^2}\left(\frac{m+1}{m-1}\right)\theta s' = \frac{1}{3}\mathrm{TTC}$$

$$\mathrm{TAS} = -\frac{2y}{R}\theta^2 s', \qquad \mathrm{TDI} = 0$$

[a] Entrance pupil is at surface.

Aberrations of a Paraboloid Mirror in Collimated Light (m=0)

- $ASA = 0$
- $ASC = \theta / (16 F^2)$
- $AAS = \theta^2 / (2F)$
- As we know, a paraboloid mirror images an on-axis object perfectly (no spherical aberration)
- The useable field size is given by coma and astigmatism
- The field size is larger for slower mirrors

Angular Aberrations of Paraboloid Mirror

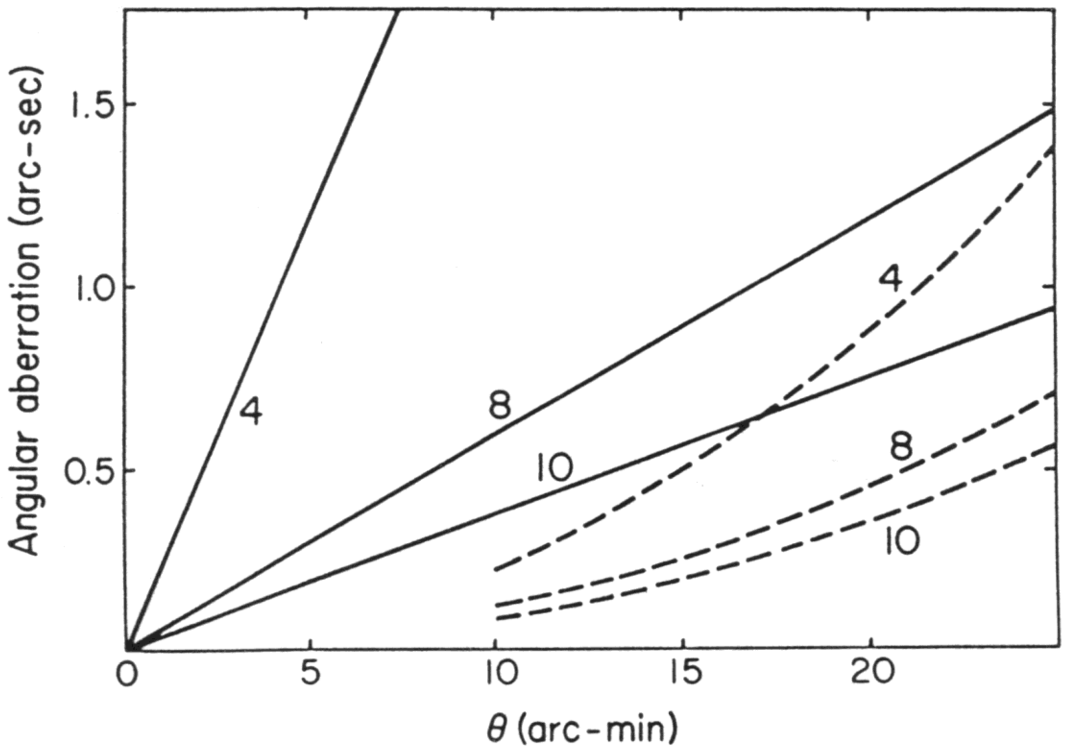


Fig. 6.1. Angular aberrations of paraboloid in collimated light at selected focal ratios. Solid lines: sagittal coma; dashed curves: astigmatism. The number on each curve is the focal ratio.

Two-Mirror Telescopes

- In the design of a two-mirror telescope, one can choose the conic constants K_1, K_2 of the primary and secondary such that there is no spherical aberration
- One solution is choosing K_1 and K_2 such that each mirror produces a perfect on-axis image
  - $K_1 = -1$ (paraboloidal primary)
  - Hyperboloidal secondary
- This is called a *Classical Cassegrain Telescope*

Aberration Coefficients for Two-Mirror Telescopes

Aberration Coefficients for Two-Mirror Telescopes with $B_{3s} = 0$[a]

$$B_{2s} = \frac{\theta}{m^2 R_1^2}\left[1 + \frac{m^2(m-\beta)}{2(1+\beta)}(K_1+1)\right] = \frac{\theta}{4f^2}\left[\;-\;\right]$$

$$B_{1s} = \frac{\theta^2}{mR_1}\left[\frac{m^2+\beta}{m(1+\beta)} - \frac{m(m-\beta)^2}{4(1+\beta)^2}(K_1+1)\right] = -\frac{\theta^2}{2f}\left[\;-\;\right]$$

$$B_{0s} = \frac{\theta^3(m-\beta)(m^2-1)}{4m^2(1+\beta)^2}\left[m + 3\beta + \frac{m^2(m-\beta)^2}{2(1+\beta)(m^2-1)}(K_1+1)\right]$$

[a] In terms of m and β, spherical aberration is zero according to the relation

$$K_1 + 1 = \frac{(m-1)^3(1+\beta)}{m^3(m+1)}\left(K_2 + \left(\frac{m+1}{m-1}\right)^2\right).$$

Angular Aberrations for Two-Mirror Telescopes

Angular Aberrations of Two-Mirror Telescopes[a]

$$\text{ASA} = \frac{1}{8}\left(\frac{y_1}{f_1}\right)^3 \left[\;-\;\right] = \frac{1}{64F_1^3}\left[\;-\;\right]$$

$$\text{ASC} = \frac{\theta}{4}\left(\frac{y_1}{f}\right)^2 \left[\;-\;\right] = \frac{\theta}{16F^2}\left[\;-\;\right] = \frac{1}{3}\text{ATC}$$

$$\text{AAS} = \theta^2\left(\frac{y_1}{f}\right)\left[\;-\;\right] = \frac{\theta^2}{2F}\left[\;-\;\right] \qquad \text{ADI} = B_{0s}$$

[a] Terms in square brackets are taken from Table 6.5 or 6.6. (previous viewgraph)

Angular Aberrations of Classical Cassegrain Telescopes

- Secondary is hyperboloid: $K_2 = -\left(\frac{m+1}{m-1}\right)^2$
- Coma is the same as for a single paraboloid
- Astigmatism is about *m* times worse, but usually still smaller than coma

$$\mathrm{ASC} = \frac{\theta}{16F^2}$$

$$\mathrm{AAS} = \frac{\theta^2}{2F}\left[\frac{m^2+\beta}{m(1+\beta)}\right]$$

$$\mathrm{ADI} = \frac{\theta^3(m-\beta)(m^2-1)(m+3\beta)}{4m^2(1+\beta)^2}$$

$$\kappa_m = \frac{2}{R_1}\left[\frac{(m^2-2)(m-\beta)+m(m+1)}{m^2(1+\beta)}\right]$$

κ = field curvature

Ritchey-Chrétien Telescopes

- The choice of the conic constants to eliminate both spherical aberration and coma gives a large useable field
- Many modern telescopes (e.g., Keck, VLT, HST) have a Ritchey-Chrétien design
- Both primary and secondary are hyperboloids
- $K_1 = -1 - \frac{2(1+\beta)}{m^2(m-\beta)} \quad , \quad K_2 = -\left(\frac{m+1}{m-1}\right)^2 - \frac{2m(m+1)}{(m-\beta)(m-1)^3}$

Ray Tracing Software

- Optical systems are usually designed with the help of ray tracing software
- These packages allow the user to define an optical system, trace rays through the optical system, and provide output for a detailed analysis
- The most commonly used ray tracing packages are Code V and Zemax

OSLO EDU

- Sinclair Optics, the developers of the OSLO ray tracing package, allow downloading of an education version from their web page
- This version is fully functional for systems with up to ten surfaces
  - This is not enough for a spectrograph or moderately complicated lens design, but sufficient to analyze most astronomical telescopes
- All you need to know can be found at http://www.sinopt.com